National Aeronautics and Space Administration

Testing GR with GWs: a reality check

arXiv:1207.4759 in a nutshell

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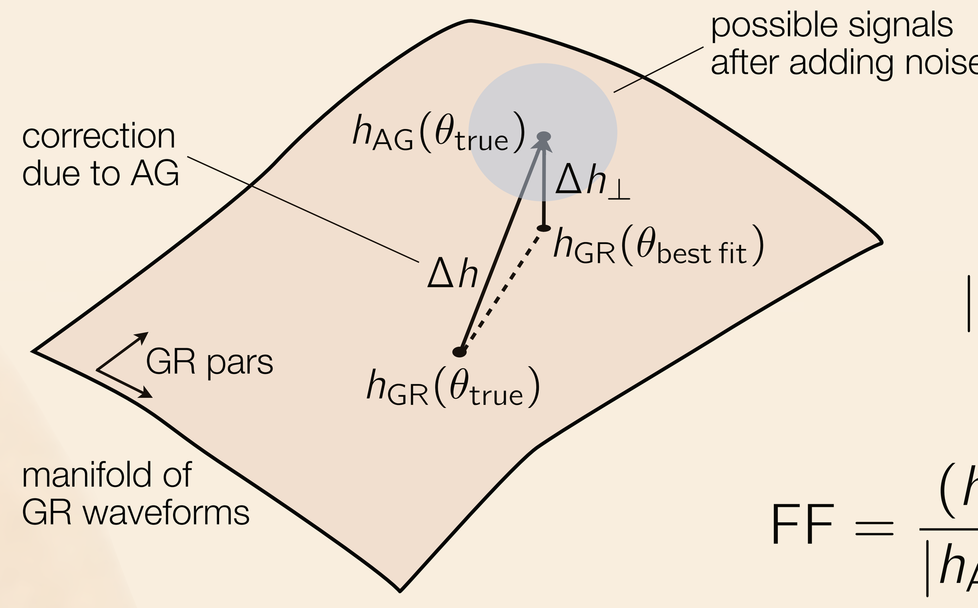
Gravitational waves from binary inspirals and other sources can be used to **test General Relativity** for self consistency and against Alternative-Gravity theories.

For most tests, and for sufficiently strong signals, there is a simple way to see how well we can do: the **SNR required for AG detection** is a simple function of the **fitting factor** between GR and AG waveforms.

For instance, 2nd-generation ground-based detectors would detect AG corrections to GR waveforms as small as 1–10% (FF=0.9–0.99).

1

Heuristically: we can distinguish Alternative-Gravity corrections when the modified waveform is sufficiently distant from the manifold of GR waveforms



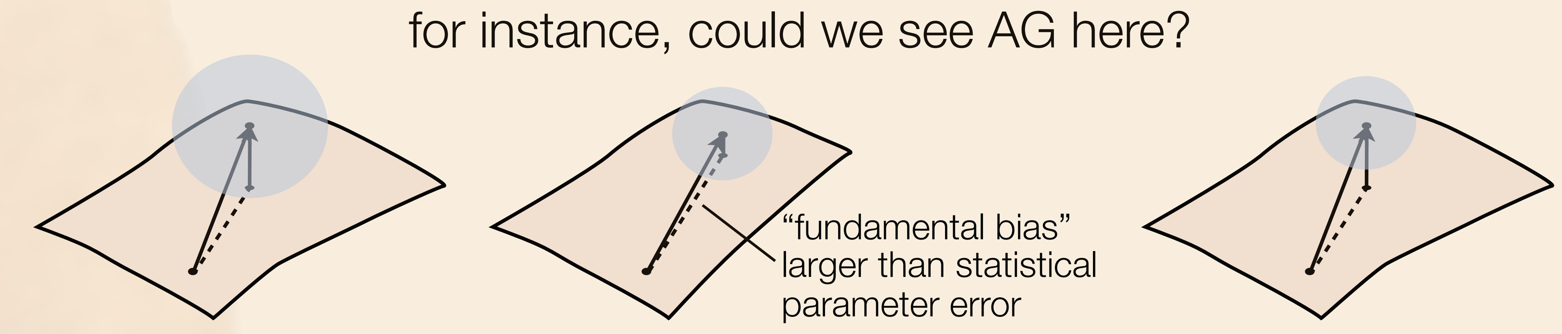
distance to closest GR waveform:

$$|\Delta h_\perp|^2 = 2|h_0|^2(1-\mathrm{FF})$$

fitting factor:

$$\mathrm{FF} = \frac{(h_{AG}(\theta_{true}), h_{GR}(\theta_{best\,fit}))}{|h_{AG}(\theta_{true})| \cdot |h_{GR}(\theta_{best\,fit})|}$$

for instance, could we see AG here?



nope, too noisy — nope, correction is absorbed by GR pars — yes, call Stockholm!

2

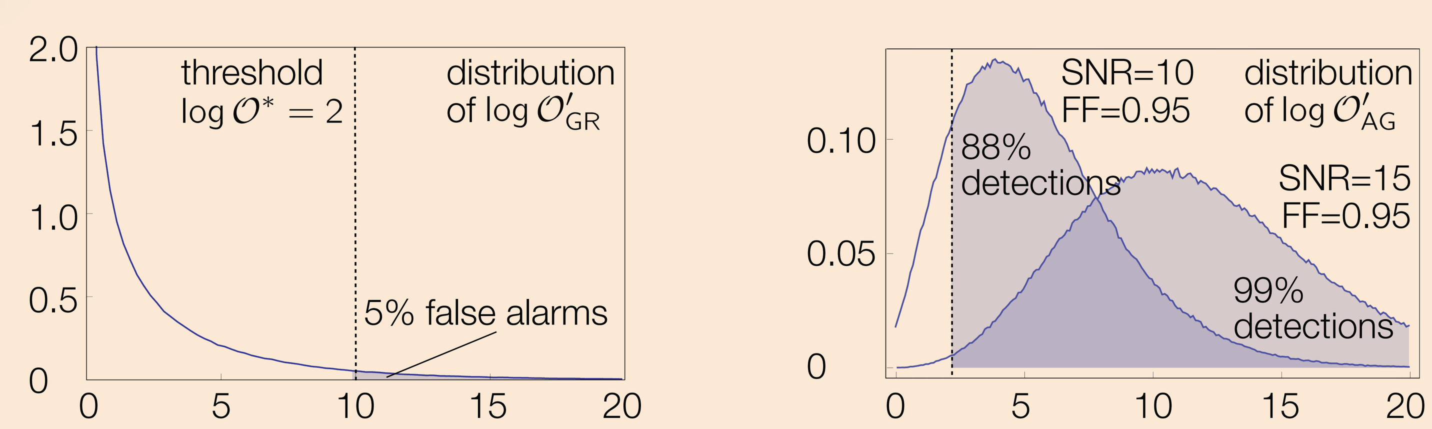
Formally: we design a decision scheme ("AG or GR?") with the Bayesian odds ratio $\mathcal{O}$ as the detection statistic; we set a threshold $\mathcal{O}^*$ and claim detection when $\mathcal{O} > \mathcal{O}^*$

model priors — parameter priors

$$\mathcal{O} = \frac{P(\mathrm{AG}|s)}{P(\mathrm{GR}|s)} = \frac{P(\mathrm{AG}) \int p(s|\theta^{i,a})\, p(\theta^{i,a})\, d\theta^{i,a}}{P(\mathrm{GR}) \int p(s|\theta^{i})\, p(\theta^{i})\, d\theta^{i}}$$

AG parameters — GR parameters

evidence (= marginal likelihood) for AG and GR models — likelihood



background: true signal is GR — detection efficiency: true signal is AG

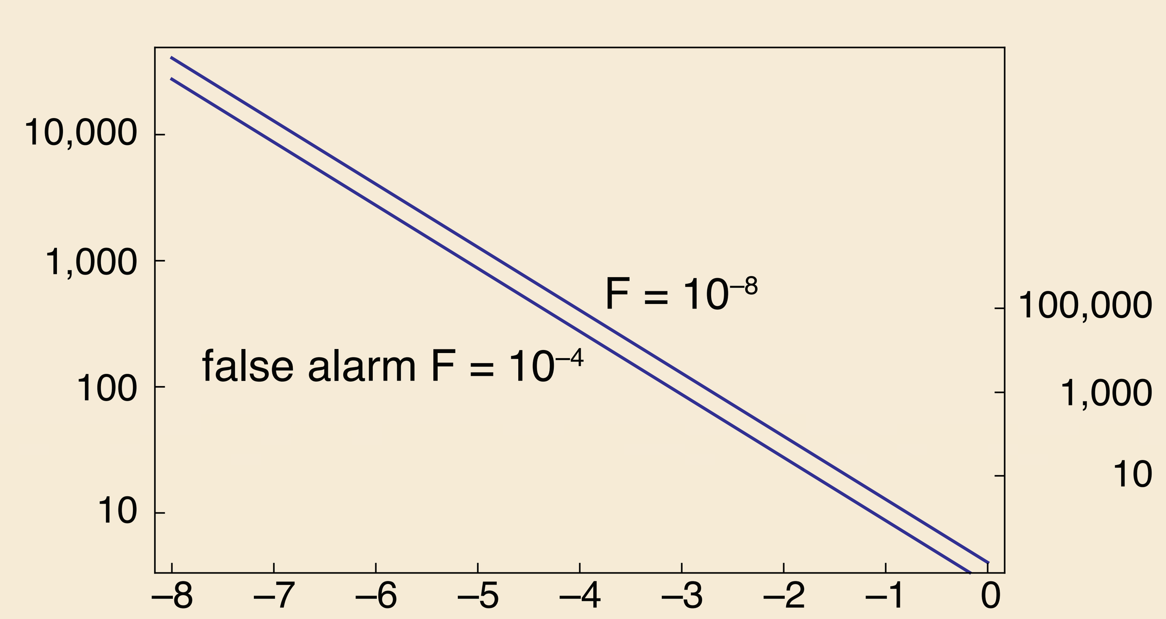
$$\mathcal{O}'_{GR} = e^{x^2/2} \qquad \mathcal{O}'_{AG} = e^{x^2/2 + x\sqrt{2(1-\mathrm{FF})}\,\mathrm{SNR} + (1-\mathrm{FF})\,\mathrm{SNR}^2}$$

renormalized odds ratios (model priors and Occam factors cancel out, see paper)

x is a normal random variable with zero mean and unit variance (a function on noise realization)

3

Practically: for strong signals, $\mathcal{O}'_{GR}$ and $\mathcal{O}'_{AG}$ are remarkably simple functions of FF and SNR alone. For a fixed false-alarm rate, we then ask what SNR yields 50%-efficient AG detection, as a function of FF



In conclusion: only very strong AG effects (FF of 0.9–0.99) would be seen in volume-limited searches, so GR tests may have to wait for 3rd-gen. ground-based or space detectors.